Synthesizing Efficient Pulses for Practical Qudit Circuits

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ABSTRACT

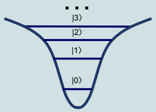
Many underlying qubit technologies have natural access to higher energy states which can expand available computational space.

Rather than considering binary two-level abstractions or qu*bits*, using 3 or 4 logical states or qu*dits* **temporarily** enables efficient circuit decompositions - reduced depth and physical device requirements.

Unfortunately, gate and coherence errors and gate durations are expected to scale unfavorably making use of these additional states challenging and demands careful engineering of **pulse level** control to minimize empirical costs.

We explore the **engineering and design of pulses** for mixed-radix quantum systems and introduce a **compiler tool-chain** to make context-dependent communication and compression choices.

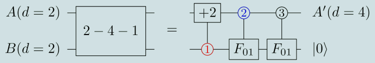
BACKGROUND



Superconducting qubits have a large spectrum of potential energy states which can be used as logical states for quantum computation.

Prior work [5] has identified circuit-level advantages of "compression" resulting in low depth circuits without need for ancilla like Toffoli and arithmetics.

Theoretically, gate durations scale quadratically [6] by using higher states which mitigates effectiveness without specialized pulse engineering and compilation.



QUDIT PULSE ENGINEERING

Quantum Optimal Control (QOC) is a technique for designing control pulses to purposefully manipulate a quantum system.

Multi-qudit drift Hamiltonian:

$$H_d = \sum_{k=1}^{Q} \left(\omega_k a_k^\dagger a_k - \frac{\xi_k}{2} a_k^\dagger a_k^\dagger a_k a_k + \sum_{l>k} J_{kl} \left(a_k^\dagger a_l + a_l^\dagger a_k \right) \right)$$

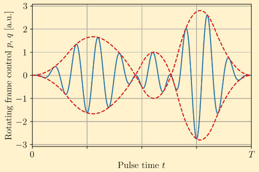
Physical parameters [7]

$$\frac{\omega_0}{2\pi} = 4.914\,\text{GHz} \quad \frac{\xi_0}{2\pi} = \frac{\xi_1}{2\pi} = 0.330\,\text{GHz}$$

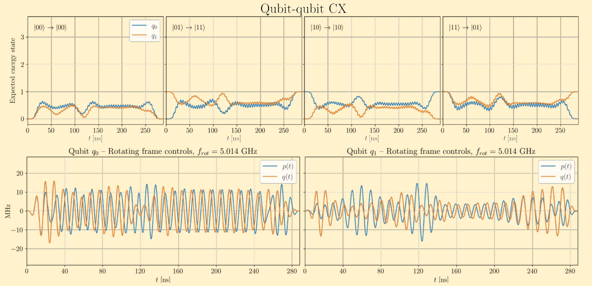
$$\frac{\omega_1}{2\pi} = 5.114\,\text{GHz} \quad \frac{J_{01}}{2\pi} = 0.0038\,\text{GHz}$$

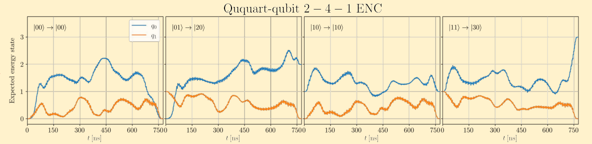
Time-dependent control:

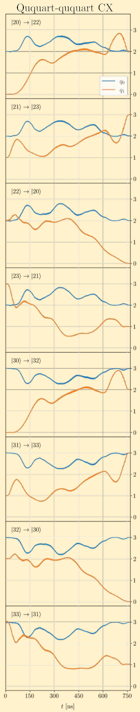
$$H_c(t) = \sum_{k=1}^{Q} f^k\left(\vec{\alpha}^k, t\right)\left(a_k + a_k^\dagger\right)$$



The goal of QOC is to find optimal control functions f^k to realize desired gates. To this end we use the software packages Juqbox [1, 2] and Quandary [3, 4], which solve the QOC problem under the RWA.
They find corresponding rotating frame controls p^k and q^k, which are parametrized with envelopes shaped by B-spline basis functions enclosing carrier waves, with coefficients $\vec{\alpha}^k$. Carrier wave frequencies are chosen to be resonant frequencies of the system.

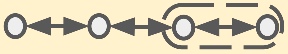




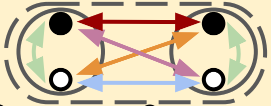


COMPILATION FOR QUDIT CIRCUITS

Example Hardware Topology



Zoomed In Ququart Interaction Topology



○ First Encoded Qubit ● Second Encoded Qubit

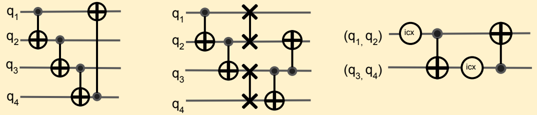
Available Ququart Compilation Gates

	Devices Used	Qudit 1 State	Qudit 2 State	CX Time (ns)	SWAP Time (ns)
→	○ ●	Ququart	-	80	64
→	○ ○	Qubit	Qubit	264	564
→	○ ○	Qubit	Ququart	-	-
→	○ ●	Qubit	Ququart	-	-
→	● ○	Ququart	Qubit	-	-
→	○ ○	Ququart	Ququart	768	-
→	○ ●	Ququart	Ququart	64+768+64	-
→	● ○	Ququart	Ququart	64+768+64	-
→	● ●	Ququart	Ququart	64+768+64	-
→	○ ○	Ququart	Ququart	n/a	1280

Compilation Constraint: Qubits can only interact when adjacent to one another, with limited connectivity must move qubits on device with minimal number of gates.

Compiling with Qudits: With increased connectivity, we can encode qubits onto the architecture and route with more flexibility, reducing the run time of the circuit.

Expanded Library: Current times are shown, and are the bare minimum for compiling on ququarts. Optimized pulses will provide more beneficial compilations.



Example Circuit **Compiled With Qubits** **Compiled on Ququarts with Encoded Qubits**

Selected References

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